1 Introduction

Welcome to the first PMC meeting of 2023! We hope you all had a restful break. Today we'll be taking a look at geometry and working with different shapes. Geometry focuses on the properties of points, lines, solids, and angles and the ways in which they are related to each other.

2 Warm-up: Background Information

Try to answer as many of these as you can!

1. A straight angle represents ____________ degrees.
2. An ____________ angle is less than 90 degrees.
3. How many edges does a cube have?
4. A full circle has ____________ degrees.
5. ____________ lines never intersect each other.
6. The ____________ is a straight line extending from the center of a circle or sphere to the surface.
7. What is a quadrilateral? Can you give some examples of shapes that are quadrilaterals?
8. A ____________ triangle is a triangle with two equal sides.
9. An ____________ angle is greater than 90 degrees but less than 180 degrees.
10. Two lines that intersect at right angles are ____________.
11. All the three sides and angles of a triangle are equal in an __________________ triangle.
12. All the angles in a triangle add up to ____________ degrees.
13. The area of a circle can be found by the formula ____________, where r stands for the radius.

3 All About Triangles

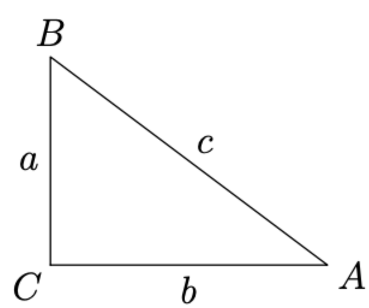
3.1 Area

To find the area of a triangle, the formula is $\frac{1}{2} \cdot b \cdot h$, where b represents the base of the triangle and h represents the height of the triangle from that base.

Try this problem: Find the height of the triangle whose base is $\frac{2}{3}$ of its height and area is 225 cm squared (Hint: Set the formula above to 225 and rewrite b in terms of h).

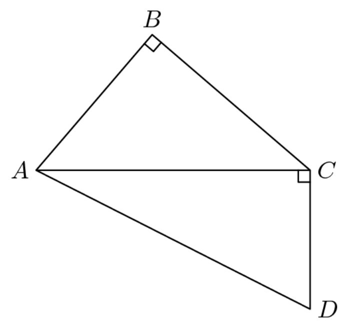
3.2 Pythagorean Theorem

A right triangle ABC, with sides a, b, and c, is shown below.



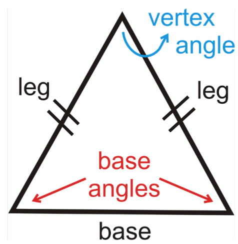
The Pythagorean Theorem states that, if C is a right angle, then $a^2 + b^2 = c^2$. The theorem also holds the other way around. **Try the problem below!**

In quadrilateral ABCD, angle B is a right angle, diagonal AC is perpendicular to CD, AB = 18, BC = 21, CD = 14. Find the perimeter of ABCD. Use the figure below to help you.



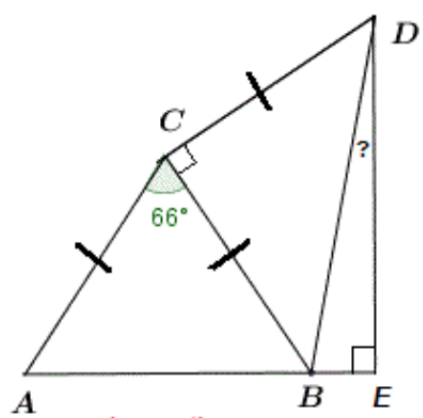
3.3 Isosceles and Equilateral Triangles

In isosceles triangles, the two angles opposite the congruent sides are also equal to each other. A diagram of an isosceles triangle is shown below.



In equilateral triangles, each angle equals 60 degrees. Another important property is that the perpendicular line drawn from the vertex of the equilateral triangle to the opposite side bisects the side into equal halves.

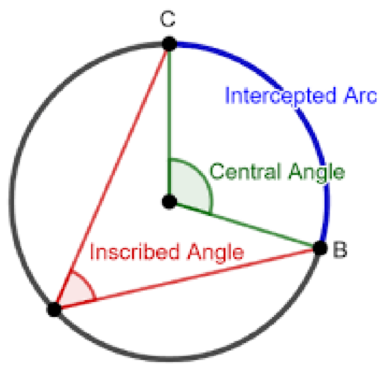
1. What is the area of an equilateral triangle with side length $8\sqrt{3}$?
2. ABC and CED are isosceles triangles. Find the size of angle BDE. Refer to the image below.



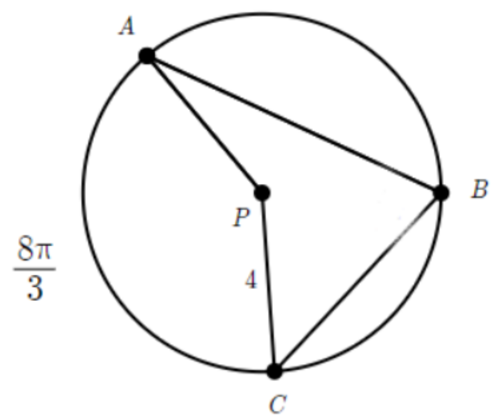
3. The perimeter of an isosceles triangle is 100 cm. If the base is 36 cm, find the length of the equal sides.
(A) 18 **(B)** 64 **(C)** 32 **(D)** 36 **(E)** 24
4. In an isosceles right-angled triangle, the perimeter is 20 meters. Find its area.
(A) 17.16 m^2 **(B)** 5.858 m^2 **(C)** 34.32 m^2 **(D)** 9.320 m^2 **(E)** 15.45 m^2

4 Circles

In circles, central angles are angles with the vertex located at the center of the circle. The measure of the central angle is the same as the measure of the arc it intercepts. On the other hand, inscribed angles are angles with the vertex located anywhere on the circumference of the circle. The measure of the inscribed angle is half of the measure of the arc it intercepts.

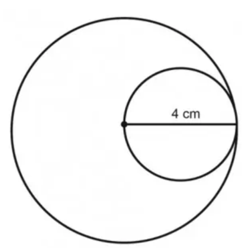


Try this problem: In the figure below, the length of $\overline{\text{PC}}$ is 4, and the length of $\overset{\frown}{AC}$ is $8\pi/3$. Find the measure of $\angle ABC$.

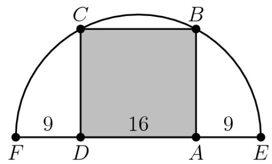


5 Challenge Problems

1. In the figure below, what is the area of the large circle minus the area of the small circle?

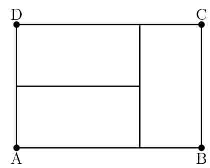


2. (2020 AMC 8 #18) Rectangle $ABCD$ is inscribed in a semicircle with diameter $\overline{FE}$, as shown in the figure below. Let $DA = 16$, and let $FD = AE = 9$. What is the area of $ABCD$?



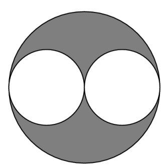
(A) 240 **(B)** 248 **(C)** 256 **(D)** 264 **(E)** 272

3. Three identical rectangles are put together to form rectangle $ABCD$, as shown in the figure below. Given that the length of the shorter side of each of the smaller rectangles is 5 feet, what is the area in square feet of rectangle $ABCD$?



(A) 45 **(B)** 75 **(C)** 100 **(D)** 125 **(E)** 150

4. (2018 AMC 8 #15) In the diagram below, a diameter of each of the two smaller circles is a radius of the larger circle. If the two smaller circles have a combined area of 1 square unit, then what is the area of the shaded region, in square units?

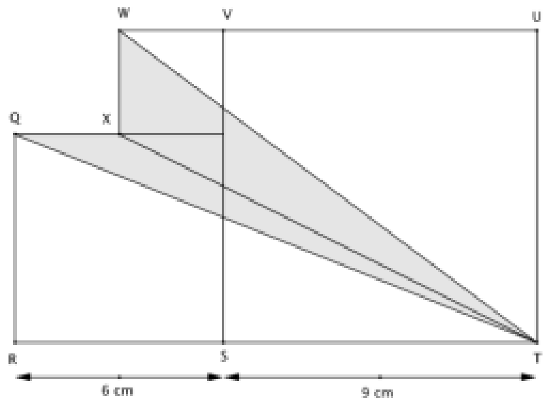


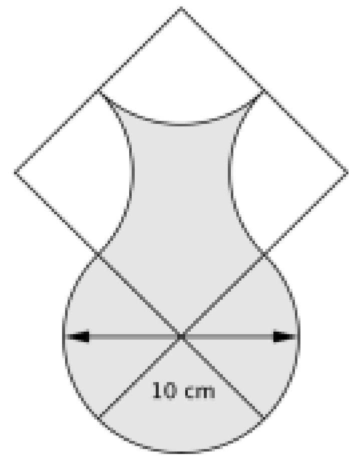
(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** 1 **(E)** $\frac{\pi}{2}$

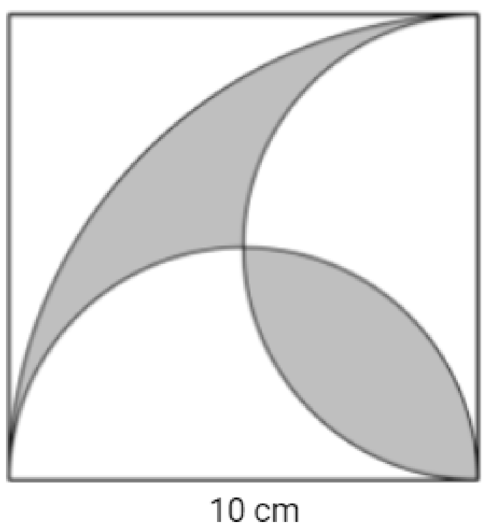
5. (2007 AMC 10A #14) A triangle with side lengths in the ratio 3:4:5 is inscribed in a circle with radius 3. What is the area of the triangle?
(A) 8.64 **(B)** 12 **(C)** 5π **(D)** 17.28 **(E)** 18

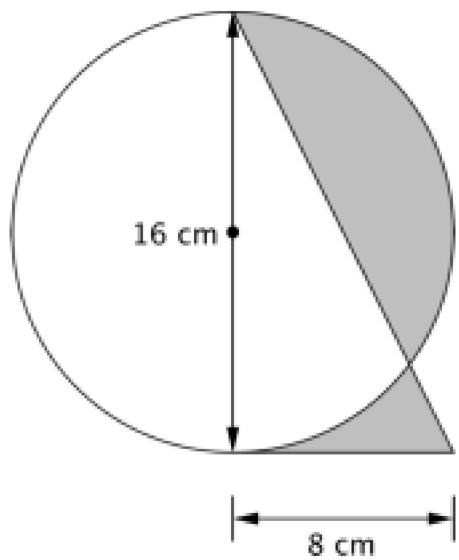
6 Area of Irregular Shapes

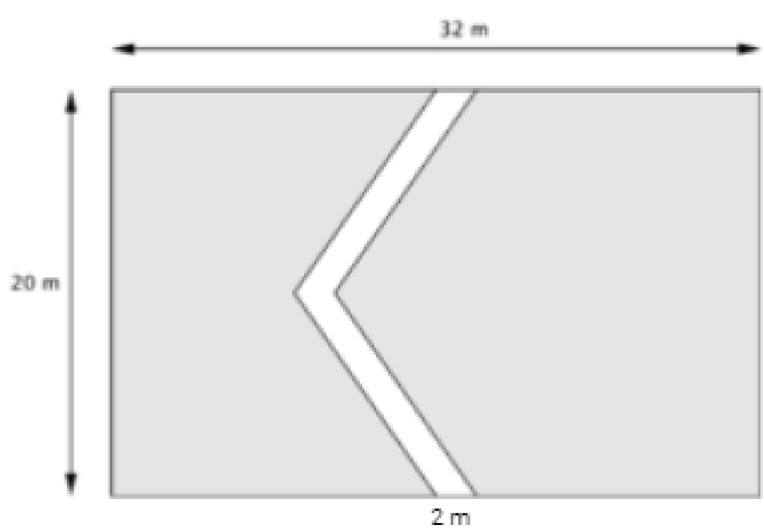
The pictures below show Devon the Square's several designs for his new house. However, Derek the triangle fires his flamethrower on parts of Devon's designs. For each design, find the area of the burnt parts of the design, which are represented by the shaded areas.


W
V
U
Q
X
R
S
T
6 cm
9 cm


10 cm


10 cm


16 cm
8 cm


32 m
20 m
2 m